Université d'Ottawa · University of Ottawa
Faculté des sciences
Mathématiques et de statistique
Faculty of Science
Mathematics and Statistics

Calculus III for Engineers
MAT 2322A - Fall 2010
Final Exam
Professor: Victor G. LeBlanc
Time limit: 3 hours. Closed books. No calculators.

Name:______________________________ ID Number:____________________

Instructions

- This exam has 15 pages and you have 3 hours to complete it.
- This is a closed book exam. Furthermore, all calculators, cell phones, pagers or any other electronic or communication devices are forbidden.
- Read each question carefully before answering.
- Questions 1 to 10 are multiple choice questions. These questions are worth 2 points each and no partial marks are possible. **Please write your answers in the corresponding boxes in the grid below entitled "Answers to multiple choice Qs".**
- Questions 11 to 16 are long answer questions and are worth 5 marks each, so organize your time accordingly. **A correct answer requires a full, clearly-written and detailed solution.** Answer each question in the space provided, using backs of pages or the extra pages at the end if necessary.
- Do not unstable the test.
- Good luck!

Answers to multiple choice Qs

1	2	3	4	5	6	7	8	9	10

Grid below is used for grading (do not write in this grid)

MCQ	11	12	13	14	15	16	Total
/20	/5	/5	/5	/5	/5	/5	/50

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1. Which of the following expressions corresponds to the integral $\int_0^8 \int_0^{y^{\frac{1}{3}}} f(x,y)\,dx\,dy$ with order of integration reversed?

A. $\int_0^2 \int_{x^3}^8 f(x,y)\,dy\,dx$

B. $\int_0^{y^{\frac{1}{3}}} \int_0^8 f(x,y)\,dx\,dy$

C. $\int_0^{y^{\frac{1}{3}}} \int_0^8 f(x,y)\,dy\,dx$

D. $\int_0^2 \int_8^{x^3} f(x,y)\,dy\,dx$

E. $\int_0^2 \int_8^{x^3} f(y,x)\,dy\,dx$

F. $\int_0^8 \int_{x^3}^2 f(x,y)\,dy\,dx$

2. If $f(x,y)$ is a differentiable function such that $\vec{\nabla} f(1,2) = \vec{j}$, only one of the following curves can be the level curve for f through the point $(1,2)$. Which one?

A. $y = 1 + x$

B. $y = 1 + e^{x-1}$

C. $y = 2e^{x-1}$

D. $y = \dfrac{2}{x}$

E. $x = 1$

F. $y = 2 + (x-1)^2$

3. Consider the parametrized curve $\vec{r}(t) = t\,\vec{i} + t^2\,\vec{j} + t^3\,\vec{k}, \quad t \in [1, 2]$. Which of the following expressions leads to the total arclength of this curve?

A. $\displaystyle\int_1^2 \sqrt{1 + 4t^2 + 9t^4}\, dt$

B. $\displaystyle\int_1^2 \sqrt{t^2 + t^4 + t^6}\, dt$

C. $\sqrt{1 + 4t^2 + 9t^4}$

D. $\sqrt{t^2 + t^4 + t^6}$

E. $\sqrt{1 + 4 \cdot 2^2 + 9 \cdot 2^4} - \sqrt{1 + 4 \cdot 1^2 + 9 \cdot 1^4}$

F. $\sqrt{(2-1)^2 + (4-1)^2 + (8-1)^2}$

4. Consider the surface which corresponds to the paraboloid $z = x^2 + y^2$ between the planes $z = 1$ and $z = 4$. Which of the following expressions leads to the total area of this surface?

A. $\displaystyle\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (x^2 + y^2)\, dy\, dx - \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (x^2 + y^2)\, dy\, dx$

B. $\displaystyle\int_{\theta=0}^{2\pi} \int_{r=1}^{2} r^3\, dr\, d\theta$

C. $\displaystyle\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sqrt{2x + 2y}\, dy\, dx - \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{2x + 2y}\, dy\, dx$

D. $\displaystyle\int_{\theta=0}^{2\pi} \int_{r=1}^{2} \sqrt{4r^4 + r^2}\, dr\, d\theta$

E. $\displaystyle\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (x^2 + y^2)\, dy\, dx$

F. $\displaystyle\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (x^2 + y^2)\, dy\, dx$

5. If C is the straight line segment starting at the point $(0,0)$ and ending at the point $(1,1)$, and $\vec{F}(x,y) = xy\,\vec{i} + y^2\,\vec{j}$, then which of the following corresponds to the value of the line integral $\displaystyle\int_C \vec{F} \cdot d\vec{r}$?

A. $\dfrac{-2}{3}$

B. 0

C. $\vec{i} + \vec{j}$

D. $\dfrac{1}{3}$

E. $\dfrac{2}{3}$

F. $\dfrac{2}{3}\,\vec{i}$

6. If $f(x,y) = x^2 + y^2$, then which of the following numbers corresponds to the global minimum value of f subject to the constraint $x^2 + 4y^2 = 1$?

A. $\dfrac{-1}{2}$

B. $\dfrac{5}{4}$

C. That global minimum value does not exist

D. $\dfrac{1}{4}$

E. 0

F. 1

7. Which of the following vector fields is conservative?

A. $\vec{F}(x, y, z) = yz\,\vec{i} + xz\,\vec{j} + xyz\,\vec{k}$

B. $\vec{F}(x, y, z) = x^2\,\vec{i} + e^y\,\vec{j} + \cos(z)\,\vec{k}$

C. $\vec{F}(x, y, z) = (x + y + z)\,\vec{i}$

D. $\vec{F}(x, y, z) = -z\vec{i} + x\vec{k}$

E. All of the above

F. None of the above

8. If S is the disk $x^2 + y^2 \leq 1,\ z = 1$ oriented upwards (i.e. unit normal parallel to $\vec{k}$), and $\vec{F}(x, y, z) = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$, then which of the following corresponds to the value of the surface integral $\displaystyle\iint_S \vec{F} \cdot d\vec{S}$?

A. 1

B. π

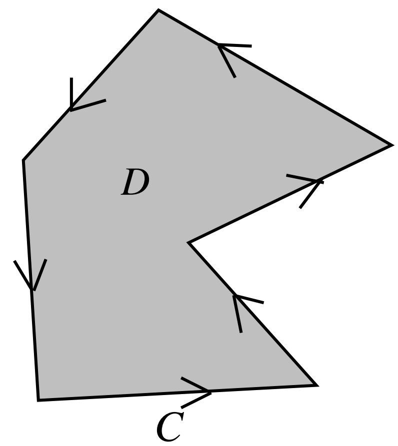
C. $\pi\,\vec{k}$

D. 2π

E. 0

F. $-2\pi\,\vec{k}$

9. Consider the two-dimensional region D drawn below, whose boundary is the oriented curve C, also drawn. Let $\vec{F}(x,y) = P(x,y)\,\vec{i} + Q(x,y)\,\vec{j}$ be a vector field with continuous partial derivatives. Then which of the following equations corresponds to Green's theorem?



A. $\displaystyle\int_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial y} - \frac{\partial P}{\partial x}\right) dx\,dy$

B. $\displaystyle\int_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial P}{\partial x} - \frac{\partial Q}{\partial y}\right) dx\,dy$

C. $\displaystyle\int_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) dx\,dy$

D. $\displaystyle\int_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} + \frac{\partial P}{\partial y}\right) dx\,dy$

E. $\displaystyle\int_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$

F. $\displaystyle\int_C Q\,dx + P\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$

10. If $z = f(x,y)$ and $x = x(u,v)$, $y = y(u,v)$, which of the following formulas corresponds to the chain rule for the partial derivative $\dfrac{\partial z}{\partial v}$?

A. $\dfrac{\partial z}{\partial v} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial u} + \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial v}$

B. $\dfrac{\partial z}{\partial v} = \dfrac{\partial z}{\partial u}\dfrac{\partial u}{\partial v} + \dfrac{\partial z}{\partial v}$

C. $\dfrac{\partial z}{\partial v} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial v} + \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial v}$

D. $\dfrac{\partial z}{\partial v} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial v}$

E. $\dfrac{\partial z}{\partial v} = \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial v}$

F. $\dfrac{\partial z}{\partial v} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial u}\dfrac{\partial u}{\partial v} + \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial v}$

11. Consider the vector field $\vec{F}(x,y) = (2xy+1)\,\vec{\imath} + (x^2+1)\,\vec{\jmath}$. Show that the vector field is conservative, and then find a scalar function $f(x,y)$ such that $\vec{F}(x,y) = \vec{\nabla} f(x,y)$. Finally, compute

$$\int_C \vec{F} \cdot d\vec{r},$$

where C is some continuous curve that starts at the point $(0,0)$ and ends at the point $(3,1)$.

12. Consider the vector field $\vec{F}(x, y, z) = 2y\,\vec{i} + 3x\,\vec{j} - z^2\,\vec{k}$.

(a) Compute the curl of $\vec{F}$, i.e. compute $\vec{\nabla} \times \vec{F}(x, y, z)$.

(b) **Using Stokes' theorem**, compute the line integral $\displaystyle\int_C \vec{F} \cdot d\vec{r}$, where C is the circle $x^2 + y^2 = 9$ in the plane $z = 1$, oriented counter-clockwise when viewed from above.

13. Consider the vector field $\vec{F}(x, y, z) = (y - x)\,\vec{i} + (z - y)\,\vec{j} + (y - x)\,\vec{k}$.

(a) Compute the divergence of $\vec{F}$, i.e. compute $\vec{\nabla} \cdot \vec{F}(x, y, z)$.

(b) Let E denote the solid cube $0 \leq x \leq 1$, $0 \leq y \leq 1$, $0 \leq z \leq 1$, and let S denote the surface which is the boundary of E, oriented outwards. **Use Gauss' divergence theorem** to compute the flux integral $\displaystyle\iint_S \vec{F} \cdot d\vec{S}$

14. Find and classify the critical points of the function $\dfrac{x^3}{3} + y^2 - 3x - 2xy$.

15. Consider the solid region E which is bounded below by the cone $z = \sqrt{x^2 + y^2}$ and bounded above by the hemisphere $z = \sqrt{4 - x^2 - y^2}$. Suppose this solid has a mass density given by $\rho(x, y, z) = (x^2 + y^2 + z^2)^3$. Find the total mass of this solid.

16. Consider the scalar function $f(x, y, z) = 4z$. Compute the surface integral $\iint_S f\, dS$, where S is the portion of the cone $z = \sqrt{x^2 + y^2}$ that lies between the planes $z = 1$ and $z = 4$.

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